Abstract

A two dimensional (2D) Eulerian multi-material radiation magneto-hydrodynamics (RMHD) code has been developed for the analysis of large deformation hydrodynamics in the presence of self-consistent magnetic fields. The code features a new Volume-Of-Fluid (VOF) based material interface tracking scheme, multi-material MHD scheme, thermal radiation transport and variety of non-linear Equation Of State (EOS) modules (both analytical and tabulated). The MHD scheme is formulated using magnetic vector potential with an implicit magnetic field diffusion calculation. The thermal radiation transport scheme used is capable of handling intense thermal radiation through both optically thin (transport limit) and thick media (diffusion limit). In this paper, we report the details of the algorithm and its application to a sample problem.

Introduction

Numerical simulation of hydrodynamics phenomena involving large material deformation, intense thermal radiation and magnetic fields is a challenging topic. A few examples are 1) laser- or thermal-radiation driven ablative acceleration, including instability growth; 2) the dynamics of Magnetic Rayleigh-Taylor (MRT) instabilities in flux compression systems driven by fusion plasma armatures1-3 3) plasma formation between the solid surfaces in electromagnetic welding systems, etc. Pure Lagrangian simulation fails when large material deformation occurs. Therefore, an Eulerian or Arbitrary Lagrangian Eulerian (ALE) scheme is necessary. The code described here is based on an ALE scheme with a remapping stage back to the initial grid at each time-step—essentially making it an Eulerian code.

RMHD model and numerical scheme

The governing hydrodynamic equations can be seen in Refs.1-3. A thermodynamically consistent multi-material hydrodynamics formulation4 along with our new volume-of-fluid (VOF) interface tracking algorithm5 is used to handle multiple materials in a mixed computational cell (a cell containing more than one material). This formulation avoids commonly used pressure relaxation methods and corresponding iterations in each mixed cell6. However, this formulation requires the evaluation of ‘M’ different energy equations in each mixed cell, where ‘M’ is the total number of materials3,4.

The VOF based material interface tracking algorithm5 consists of three different phases: Interface reconstruction, Lagrangian deformation of material interfaces and finally the Eulerian advection. A Piecewise Linear Interface Construction (PLIC) scheme is used in the first phase of VOF algorithm where the interface is represented by a straight line in a cell. The slope of this interface is obtained from the gradients of volume fraction of each material. Different materials contained in a mixed cell are ordered using a dynamic layered material ordering scheme5. The third stage of VOF scheme is material advection. This is done using a second order accurate Monotonic Upwind SCheme (MUSCL)2. The mass advected in the Eulerian advection step is calculated using a characteristic trace back scheme5 based on the area of intersection between the old and new meshes.
For the simulation of MHD flows, the electromagnetic field components are required to be updated self consistently with the fluid flow and external electric circuit equations\(^1\). This has been achieved using our new MHD algorithms described in Refs\(^1,2\). Magnetic diffusion into electrical conductors is solved implicitly using the magnetic vector potential\(^2\). During the advection/remap step, the magnetic field components are advected using a 2\(^{nd}\) order MUSCL scheme.

For practical cases, the material velocity is typically smaller than the speed of light. Therefore, at any instant of time, the radiation field can be regarded as quasi-static. Furthermore, we have used the frequency averaged scheme, i.e., “grey” approximation. The radiation transport equation\(^7\) is solved in 2D by using a numerical scheme described by Ramis et al\(^7\). The radiation energy is coupled to material through a Symmetric Semi-Implicit (SSI) scheme\(^7\).

**Application to a sample problem**

The algorithm developed has been validated against known analytical solutions of several benchmark problems\(^2,5\). Here, we present the results of Magnetic Rayleigh-Taylor (MRT) instability analysis in a direct energy conversion system which converts Inertial Fusion Energy (IFE) plasma kinetic energy into pulsed electrical energy\(^1-3\). A schematic of the proposed Magnetic Flux Compression (MFC) system driven by fusion plasma sphere is given in Fig. 1. Preliminary studies\(^1\) show that the proposed MFC system is promising in terms of overall conversion efficiency. However, the plasma expanding across the magnetic field is subject to MRT instabilities.

In the sample problem, the initial plasma kinetic energy and mass were taken as 280 MJ and \(-4.4\) mg, respectively, with temperature of 100 keV (fusion-driven) and density of \(-10^{-6}\) kg/m\(^3\). The initial magnetic field was \(-5\) Tesla. The coil radius and length were 1.5 m and 4.5 m, respectively.

Fig. 2 shows the spatial variation of plasma density. Starting with a uniform density plasma sphere, the plasma evolves into a shell-like geometry near the stagnation time, where the outer surface slows down due to magnetic pressure and the inner region catches up with the outer surface. The thickness of this region is \(-0.05-0.1\) m with an average temperature of \(-30\) keV. The electrical conductivity of the plasma near the surface is sufficiently high \(-10^9\) S/m to prevent magnetic field diffusion. That is, even at larger expansions, the diffusion of magnetic field into the plasma is found to be negligible.

Towards the stagnation time, we have observed the evolution of MRT instabilities on the surface of the plasma even with an unperturbed initial plasma state, as shown in Fig. 3. Therefore, the analysis is repeated.

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Fig. 1: Schematic showing MFC by expanding plasma sphere (not to scale).

Fig. 2: Spatial variation of plasma density (scaled by \(10^6\) kg/m\(^3\)) at stagnation time.

Fig. 3: Zoomed in plot of plasma vacuum interface near the stagnation time indicate the development of MRT instability.
for plasmas with an initial sinusoidal perturbation of different wavelengths $\lambda$ and amplitudes $\alpha$. This helps us identify the critical $\alpha$ and $\lambda$ which produce significant degradation in system performance.

The instability amplitude growth obtained for different cases of initial perturbation $\alpha$ and $\lambda$ are summarized in Fig. 4. The perturbation amplitude continues to grow exponentially with nearly constant growth rate ($\gamma$) and makes a transition into a nonlinear phase towards the stagnation time. Hence the amplitude growth of modes towards stagnation time, although exponential in nature, is lower than the growth predicted by linear theory. We also note that extremely large flute structures and plasma jetting, which could damage the coil/cavity wall, is not observed for $\alpha_{in} < \lambda_{in}/10$. However, for $\alpha_{in} > \lambda_{in}$, the instability amplitudes are large enough, especially for longer $\lambda$ modes, to cause plasma jetting leading to significant reduction in flux compression efficiency, see Fig. 5. In order to quantify the decrease in efficiency ($\varepsilon$), we have shown the variation of $\varepsilon$ for different values of mode number and $\alpha_{in}$ in Fig. 6. In general, a loss of efficiency $\sim 20\%$ is expected for longer $\lambda$ modes ($n<20$) and short $\lambda$ modes ($n>20$) when $\alpha_{in} - \lambda_{in}$ and $\alpha_{in} - 2\lambda_{in}$ respectively.

Conclusions

A two-dimensional (2D) RMHD code has been developed and validated. The validated algorithm has been applied to study MRT instabilities in a MFC system driven by expanding high-pressure IFE plasma.

References