An interpretation of the Cosmological Constant from the Physical Constants

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Abstract

The simplest formulation of the present Universe is given by the L -Cold Dark Matter (L -CDM) model, where L being referred to as cosmological constant. In this model, the dominant energy budget of the Universe is described by the cold dark matter and the dark energy. The gravitational behavior of the dark energy for the expanding Universe is attributed to L which is equivalent to the vaccum energy density. The discovery of accelerated expansion of the Universe has opened the most important window not only in modern cosmology but also for the fundamental physics. In this work, we perform some analytical calculations to estimate the value of L using the results from the cosmological observations. The numerical value of L determined from cosmological observations is found to be very small, but it represents late time accelerated expansion of the Universe. Acceleration of the expanding Universe due to L started dominating at the epoch when age of the Universe was ~ 70% of the current age. We also discuss the unique relationship between L and various constants of physics derived using axiomatic approach. The numerical value of L derived from the physical constants is in good agreement with the value obtained from cosmological observations.

Keywords: Cosmological Constant, Dark Energy, Physical Constants, Planck Scale

Introduction

After the discovery of accelerated expansion of the Universe in 1998 from the supernovae observations [1, 2], dark energy has become one of the most attractive area of research in modern cosmology. The energy density associated with the dark energy gives rise to a negative pressure and asymptotically approaches to a constant known as cosmological constant (L). This cosmological constant plays an important role in explaining the observed accelerated expansion of the Universe. The dark energy through L is observed to contribute ~ 70% to the present cosmic budget [3]. About 26% of the cosmic budget is attributed to the cold dark matter (CDM) and rest 4% is the ordinary matter and radiation predicted by the Standard Model of partcle physics [3]. The current expansion rate of the Universe is determined by the value of Hubble constant H_0 = 70 km s^{-1} Mpc^{-1} at present epoch. This simple explanation of the present day Universe is given within the framework of so called L -CDM model in the modern cosmology. This model is observed to be the simplest theoretical description of the large scale evolution of the Universe as well as the astrophysical observables. However, the exact physical nature of the dark energy and constituents of the dark matter have not been clearly understood until now. A huge discrepancy is observed between the measured and theoretically estimated values of L [4]. Bound dark energy models have been proposed as an alternative to the L -CDM model for explaining the dynamics of the expanding Universe and to understand the nature of dark energy using extention of the Standard Model of particle physics [5]. Fundamental hypothesis of the standard cosmology states that the basic laws of physics remain same throughout the Universe. However, according to Dirac, constants of nature are not universal constants but they reflect the state of the Universe [6]. This indicates a possible spatial and temporal variation of the physical constants. This hypothesis motivates for investigation of speculated space-time dependence of the physical constants which can lead to a new physics. Interestingly, L can be expressed in terms of a few constants of physics. In this work, we investigate the relations between L and some constants of physics. The paper is organized as following: the constants of nature which are commonly used in astrophysics and cosmology are discussed in Section 2. In Section 3, the cosmological constant (L) is described. The relationship between L and the physical constants is discussed in Section 4. Finally, we discuss and conclude the results in Section 5.

Physical Constants

The basic laws of physics are constructed using a few constants like the speed of light in vaccum c, Planck constant h, Newtonian gravitational constant G, charge of the electron e and Boltzmann constant k_B. Combinations of these physical constants define natural units of measurement called *Planck Units*, whose magnitudes define the *Planck Scale*. The physical units at Planck scale play a very important role in cosmology to describe the early Universe and fundamental physics. The set of units at *Planck scale* are defined as follows:

- Planck length: $L_p \equiv \sqrt{\frac{G\hbar}{c^3}} = 1.6 \times 10^{-35}$ m
- Planck time: $t_p \equiv \sqrt{\frac{G\hbar}{c^5}} = 5.4 \times 10^{-44}$ s
- Planck mass: $M_p \equiv \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-8}$ kg
- Planck temperature: $T_p \equiv \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} = 1.4 \times 10^{32}$ K

where $\hbar = (\frac{h}{2\pi})$ is the reduced Planck constant. Planck length represents a lower limit to the proper distance in any space-time and quantum gravity effects become dominant at this scale hinting for search of new physics [7]. No physical instrument can be designed to measure the distances smaller than Planck length. Planck time is used to describe the sequence of events in the early Universe after the big-bang and also for the timescales associated with the quantum gravitational effects. Planck mass is used to define the primordial black holes formed in the early phases of the Universe. From the big-bang theory, the Universe has originated from very hot plasma with temperature equivalent to Planck temperature. Another important physical constant defined in the spectroscopy is the

Fine Structure Constant (α). In terms of the physical constants, it is expressed as $\alpha \equiv \frac{e^2}{\hbar c}$ and its numerical value is measured to be 0.0072973525693 with very high precision.

Cosmological Constant

The cosmological constant (Λ) was introduced by Einstein in cosmology to derive the theory of a static Universe [8]. Einstein's equation for a real Universe represents relation between the curvature of spacetime and the distribution of its components and is given by:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + g_{\mu\nu}\Lambda \tag{1}$$

where $G_{\mu\nu}$ is the Eienstein's gravitational field strength tensor, $T_{\mu\nu}$ is the energy momentum tensor which describes the distribution of the matter component in the Universe, Λ represents the dark energy component of the Universe, and $g_{\mu\nu}$ is the metric tensor in the general theory of relativity. The physical interpretation of the Einstein's equation is *Space-time tells matter how to move; matter tells space-time how to curve.* Einstein's gravitational field equation is derived from an action and extention of the gravitational action by entering Λ does not introduce any additional degree of freedom. Λ in Equation (1) can be interpreted as energy momentum tensor for vacuum (perfect fluid) in the Universe. The energy density associated with Λ is referred to as *vacuum energy density or dark energy density* (ϵ_Λ) and it can be defined as:

$$\epsilon_\Lambda = \frac{c^4}{8\pi G}\Lambda \tag{2}$$

The mass density (ρ_Λ) corresponding to the vacuum energy density is expressed as:

$$\rho_\Lambda = \frac{\epsilon_\Lambda}{c^2} \tag{3}$$

The time-component (μ = 0; ν = 0) of Einstein's equation for a flat Universe consisting of perfect fluid with non-interacting components gives Friedmann equation which is given by:

$$\frac{1}{a^2(t)}\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3c^2}(\epsilon_m + \epsilon_\Lambda) \tag{4}$$

where $a(t)$ is a dynamical variable known as the cosmological scale factor which determines the expansion of Universe and ϵ_m is the energy density associated with matter present in the Universe. Assuming that the Universe is dominated by the dark energy associated with Λ, dynamics of expanding Universe is given by solving Friedmann equation (Equation (4)) for $\epsilon_m \approx 0$. In this case, the scale factor is found to be:

$$a(t) \propto exp(c\sqrt{\Lambda/3}\, t) \tag{5}$$

This indicates the expansion of the Universe for positive and nonzero cosmological constant (Λ = 0). Dimensional arguments suggest that the dimensionality of Λ should be inverse-square of distance [L^{-2}]. The acceleration of the expanding Universe is given by :

$$\frac{d^2a}{dt^2} = c^2\frac{\Lambda}{3}a(t) \tag{6}$$

Therefore, Friedman equation derived from Einstein's field equation with Λ determines the evolution of the scale factor $a(t)$ which describes the accelerated expansion of the Universe. Also, the dark energy component of the Universe represented by Λ can be characterized as a perfect fluid with high negative pressure (p_Λ) which causes the accelerated expansion of the Universe. Hence the equation of state for dark energy should satisfy the following relation [9]:

$$p_\Lambda = -\epsilon_\Lambda \tag{7}$$

where ϵ_Λ is given by Equation (2).

The spatial component (μ = 1; ν = 1) of Einstein's equation (Equation (1)) for a flat Universe filled with matter and dark energy gives the second Friedmann equation:

$$\frac{2}{a(t)}\frac{d^2a}{dt^2} = -\frac{8\pi G}{c^2}(p_m + p_\Lambda) - \frac{8\pi G}{3c^2}(\epsilon_m + \epsilon_\Lambda) \tag{8}$$

where p_m is the pressure due to matter component of the Universe. Assuming that the pressure due to matter is negligible ($p_m \approx 0$) and using Equation (7) in Equation (8) we get:

$$\frac{2}{a(t)}\frac{d^2a}{dt^2} = \frac{8\pi G}{3c^2}(2\epsilon_\Lambda - \epsilon_m) \tag{9}$$

For the static Universe, rate ($\frac{da}{dt}$) and acceleration ($\frac{d^2a}{dt^2}$) both should be zero. Therefore, if $\frac{d^2a}{dt^2} = 0$ Equation (9) reduces to

$$\epsilon_\Lambda = \frac{\epsilon_m}{2} \tag{10}$$

Therefore, if we set $\epsilon_m = \frac{c^4}{4\pi G}\Lambda$ the set of Friedmann equations can describe the Einstein's static model of the Universe. In the context of Einstein's static Universe, it is also argued that Λ should be zero because the elements of space-time geometry depend on the energy density and pressure in the Universe [10]. In this case, the observations interpreted in terms of the accelerated expansion of the Universe can alternatively be interpreted in terms of the inhomogeneity of the Universe or extinction of light from the distant supernovae [11, 12, 13]. Also, the total energy estimation of Einstein's static homogeneous and isotropic Universe requires an infinite radius (spatially flat Universe) which results in zero cosmological constant [14]

For a flat Universe, the *critical energy density* at present epoch is defined as:

$$\epsilon_{c,0} = \frac{3c^2}{8\pi G}H_0^2 \tag{11}$$

If the energy density is greater than $\epsilon_{c,0}$,the Universe has positive curvature and if the energy density is less than $\epsilon_{c,0}$, the Universe will be negatively curved. The numerical value of $\epsilon_{c,0}$ is estimated to be ~8.3 x 10^{-10} Joule m^{-3} and the corresponding *critical mass density* $\rho_{c,0} \sim 9.2 \times 10^{-27}$ kg m^{-3}. In cosmology, the energy density of the Universe is often expressed by a dimensionless *density parameter* (Ω) which is defined as the ratio of energy density to the critical density. Therefore, for dark energy component of the Universe associated with Λ , the density parameter is given by:

$$\Omega_\Lambda = \frac{\epsilon_\Lambda}{\epsilon_c} \tag{12}$$

where Ω_Λ is one of the parameters in the Λ-CDM model. From the discovery of accelerated expansion of the Universe in 1998 using supernoave observations, the current value of Ω_Λ is found to be ~ 0.73 [1, 2]. Using this observed value of Ω_Λ and $\epsilon_{c,0}$ in Equation (12), the observed value of Λ is estimated to be $\Lambda_{obs} \approx 1.25 \times 10^{-52}\, m^{-2}$. Therefore, the cosmological observations suggest a positive and very small value of cosmological constant for the dark energy component of the Universe.

Relation between Cosmological Constant and Physical Constants

Assuming that the modern cosmology with Λ provides satisfactory explanation of the

Universe, two important *Cosmological scales* can be defined in the theory of gravity for cosmology. From the dimensional analysis, two physical units at *Cosmological scales* are defined as:

- Length scale: $L_\Lambda \equiv \frac{1}{\sqrt{\Lambda}}$
- Time scale: $t_\Lambda \equiv \frac{1}{c\sqrt{\Lambda}}$

The above *cosmological scales* are useful in understanding the present Universe. From the observed value of cosmological constant $\Lambda_{obs} \approx 1.25 \times 10^{-52}\, m^{-2}$ (Section 3), cosmological length and time scales are estimated to be $L_\Lambda \approx 8.7 \times 10^{25}\, m$ and $t_\Lambda \approx 9.2 \times 10^{9}$ years respectively. This indicates that the cosmological constant can influence the accelerated expansion of the Universe on large distance scales $L_\Lambda \sim 10^{25}\, m$ which is about 66 % of the Hubble radius ($\frac{c}{H_0} = 1.32 \times 10^{28}\, m$) of the present Universe. Similarly, the cosmological timescale $t_\Lambda \approx 9.2 \times 10^{9}$ years corresponds to about 70 % of the Hubble time ($\frac{1}{H_0} = 13.4 \times 10^{9}\ years$) which represents the current age of the Universe to a good approximation. Therefore, the numerical value of cosmological timescale determined from Λ_{obs} suggests late time accelerated expansion of the Universe.

Theoretical Estimation for L

Comparison of cosmological length (L_L) and time (t_L) scales with the corresponding *Planck scales* (Section 2), which are used to describe the early Universe, a huge discrepancy in their 6 numerical values (~10^{60}) is observed. Since L is associated with the fluctuations in the *vacuum energy density*, it can be theoretically derived only in the framework of quantum field theory. However, theoretical properties of L can be investigated from the dependence of the cosmological constant on the physical constants using axiomatic approach [15]. The four axioms for the theoretical cosmological constant (L_{theo}) are based on the following physical contents:

(I) L_{theo}theo should only depend on the physical constants in nature.

(ii) L_{theo} should have a lower bound.

(iii) Dependece of L_{theo} on the physical constants should be described in the simplest way.

(iv) Relation between L_{theo} and the physical constants should be invariant under any transformation.

On the basis of above four axioms, an explicit expression for L_{theo} is given by [15]

$$\Lambda_{theo} = \frac{G^2}{\hbar^4}\left(\frac{m_e}{\alpha}\right)^6 \qquad (13)$$

where m_e is mass of the electron (which is also a constant). Using the numerical values of the physical constants, we get $\Lambda_{theo} \approx 1.34 \times 10^{-52}\, m^{-2}$ which is consistent with the observed value of the cosmological constant (Λ_{obs}) determined from the observations as discussed in Section 3. Therefore, corresponding values of vacuum energy density (defined in Equation (2)) are in good agreement. Thus, the cosmological constant introduced by Eienstein in the gravitational field equation can be derived as a function of the physical constants using a set of physical axioms similar to information theory [16].

Effect of variations in Physical Constants on L

Dirac's hypothesis leads to the possibility of space-time variation of fundamental constants of nature [6]. Grand-Unification Theories (GUTs) in general relativity predict that Newton's gravitational constant (*G*) is not a universal constant but it varies as a function of low mass dynamical scalar fields [17]. From the dynamics of the Earth-Moon system, an upper-limit on the time variation of G is obtained to be [18]

$$\frac{1}{G}\frac{dG}{dt} = (0.2 \pm 0.7) \times 10^{-12}\ year^{-1} \qquad (14)$$

Some cosmological observations also provide constraints on the above value [19]. From the observation of absorption features in the optical spectra of distant quasars, evidence for cosmological evolution of (i.e redshift dependence) is reported in the literature [20, 21, 22]. In the redshift range z = 0.5 - 3.5, the time variation of fine structure constant is found to be [21, 22]:

$$\frac{\alpha - \alpha_0}{\alpha_0}(= -0.72 \pm 0.18) \times 10^{-5} \qquad (15)$$

where α_0 is the value of fine structure constant at present epoch ($z = 0$). This indicates that fine structure constant at present (α_0) has value greater than its past value (α) at nonzero redshift ($z = 0$). Modified gravity theories also predict time dependent variation of α which can break the equivalence principle in electrodynamics [23]. The equivalence principle in general theory of relativity stipulates that the gravitational acceleration of any object is independent of its mass. At *Planck scales*, the quantum gravity effects are speculated to be responsible for the Lorentz Invariance Violation (LIV) in the theory of relativity [24]. The LIV can introduce small correction to the speed of light in vacuum c at Planck energy scale ~ 10^{19} GeV. At *Cosmological scales*, matter or gravitational effect of the Universe may modify the value of c. The space-time variations of the physical constants lead to the coupling between the matter component of the Universe and massless fields [17]. Thus, the constants like G, c and α may not have universal values in the cosmological constext. Therefore, the relationship between L and physical constants given by Equation (13) suggests that the cosmological constant should be time or redshift dependent. Time or redshift dependence of L is being explored through α-measurements using cosmological observations at different redshifts [25]. Models with varying L indicate presence of non-interacting dark energy and warm dark matter in the Universe [25]. Whereas, in L-CDM model, the dark energy is characterized by constant L and the dark matter is cold. Recently, the combination of multiple cosmological observations have been used to constrain the equation of state of dark energy and its energy density in the Universe [26]. In this multiprobe survey, a Universe with no dark energy component is ruled out and the geometry is found to be consistent with the spatially flat Universe. In another study, the phenomenological single-parameter dark energy parametrizations favor a phantom equation of state at present epoch [27]. A thorough investigation of dark energy using various cosmological probes is still required to understand the nature of dark energy.

Discussion and Conclusion

The dynamics of accelerating Universe indicates presence of a new component known as dark energy which dominates the late time expansion. The Universe can be described by two fundamental scales known as *Planck* and *Cosmological scales* defined by the physical constants and cosmological constant respectively. The astrophysical observations in modern cosmology show good agreement with the cosmological constant (L) interpretation of the dark energy in the L-CDM model. Modern cosmology describes the structure formation and expansion dynamics of the Universe using physical models like L-CDM model. In such models, fundamental laws of physics play an integral role to explain the results of observational cosmology. A nonzero and positive value of L along with other cosmological parameters helps in understanding the slowly decaying nuclei within the current age of the Universe (~ 13.4 x 10^9 years) and life of old cosmic objects like white dwarfs. Any temporal or spatial deviation of L in general theory of relativity will lead to a new physics [28]. The *Planck scale* derived using the physical constants have important relevance for the quantum theory of gravity or simply quantum gravity. The description of L using various physical constants also strongly supports the interpretation of the dark energy in the modern observational cosmology. The investigation of the constancy of the physical constants provide strong motivation for new physics in modern cosmology.

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