Chapter 17

Magnetic Systems for Vacuum Tube Devices

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There are requirements of magnetic field for vacuum devices like Relativistic Magnetron and Backward Wave Oscillator (BWO). Such field can be generated by permanent magnet, or electromagnets. This chapter discusses design approach of electromagnets and energy storage capacitor for generation of pulse magnetic field. First part discusses the design of magnetic coils, while analysis of capacitor bank is given in second part.

17.1. Design of Magnetic Coil

Design approach of the magnet coil is based on two objectives;

(i) To achieve required magnetic field in a given geometrical constraints.

(ii) To minimize coil size and resistive losses.

All the parameters of magnetic coil and their design basis are explained as follows.

Material: Low resistivity, high current density and economic viability are the desired quality. There are no much options for it, and copper is natural criteria. For high magnetic field and long duration, super conductor may also be considered.

Cross section: For a given current density and ampere turn, resistive losses are inversely proportional to square of cross section of the conductor. Hence it should be as high as possible. Cross section is limited by skin depth or machining.

Current: Even if, resistive losses are indifferent with current density for a given ampere turn, it effects size of the coil. Higher the current density, lower the mass. Higher current density results into lower inductance, which improves the rise time of the coil. Current density is limited by thermal criteria. And current is limited by current density for a given cross section.

Inner radius: It should be as low as possible to reduce the size of the coil. Magnetic field at the axis of circular coil decreases with its radius. To compensate it, more turns have to be added, which results into bulky coil. Minimum value of the inner radius is decided by the geometry over which it is to be placed.

Horizontal turns: Magnetic coils are to be wound in many layers to obtain require field. To reduce the layers, coils turns should be kept as close as possible in a given layer. Number of turns in each layer is limited by length of the coil.

Insulation between layers: Voltage across the coil, distributes nearly same to all the turns. There must be insulation to sustain the potential difference between these layers. Optimization of layers and total turns require iterations, which are discussed in following part of the chapter.

Optimization of number of turns: It requires several iteration and need a computational tool to compute magnetic field for a given coil configuration. Magnetic field of a coil formed in a circular configuration can be computed by an algorithm explained as follows.

Every coil of circular configuration can be split in number of coaxial circular coils equal to total number of turns as shown in Figure 17.1.

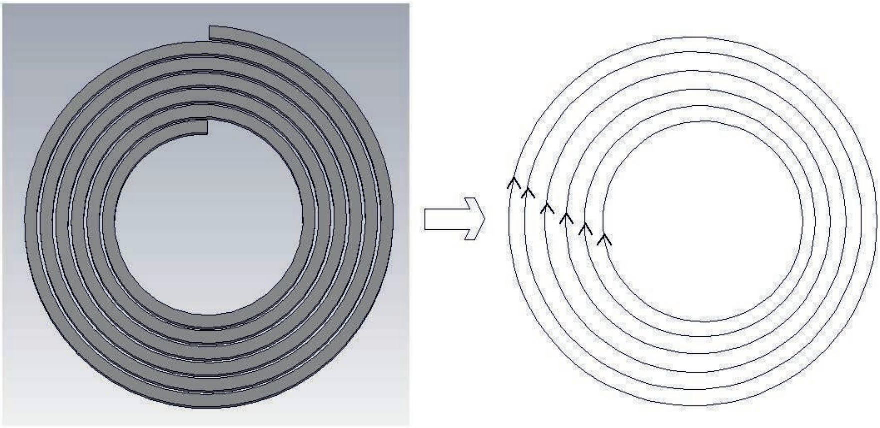

Figure 17.1. simplification of multi turn coil.

Magnetic field of full coil is superposition of the all circular coils. Magnetic field of circular coil is given by Eq. (17.1) to Eq. (17.3), which are derived from the Biot Savart law. Figure 17.2 shows the circular coil.

$$B_X = \frac{\mu_o.I.r.Z}{4.\pi}\int_0^{2\pi}\frac{\cos\theta}{[x^2+y^2+z^2+r^2-2.r(x.\cos\theta+y.\sin\theta)]^{1.5}}.d\theta \tag{17.1}$$

$$B_y = \frac{\mu_o.I.r.Z}{4.\pi}\int_0^{2\pi}\frac{\sin\theta}{[x^2+y^2+z^2+r^2-2.r(x.\cos\theta+y.\sin\theta)]^{1.5}}.d\theta \tag{17.2}$$

$$B_z = \frac{\mu_o.I.r}{4.\pi}\int_0^{2\pi}\frac{r-y.\sin\theta-x.\cos\theta}{[x^2+y^2+z^2+r^2-2.r(x.\cos\theta+y.\sin\theta)]^{1.5}}.d\theta \tag{17.3}$$

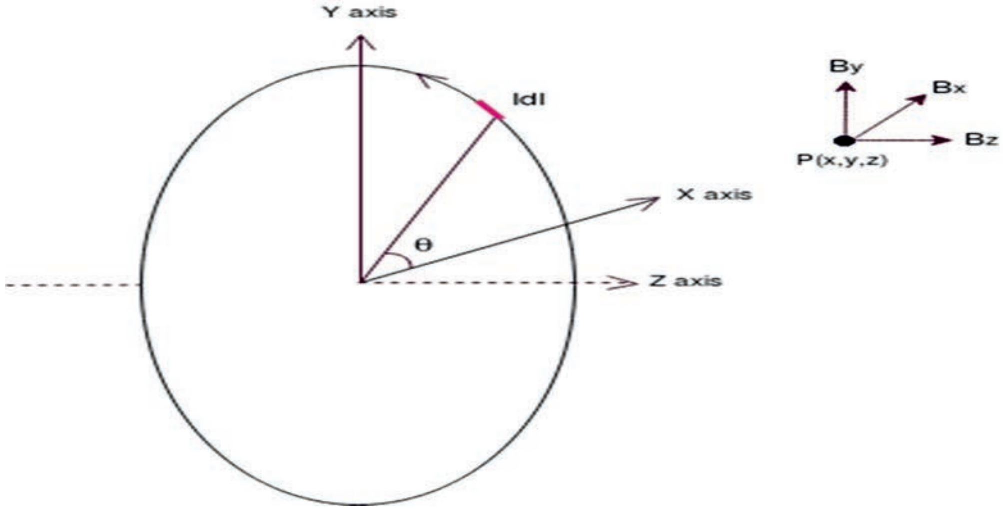

Figure 17.2. Magnetic field due to a circular coil

Ampere turns can be estimated by magnetic field formula for solenoid coil. Magnetic field gets reduced significantly at ends for finite solenoid. Hence additional turns are needed to be added at the ends to improve the magnetic field profile and it improves with iterations only.

For computing magnetic field of helical coil, it can be considered as two groups of semi circular coils as shown in Figure 17.3. Both the groups have different axis and radius than full coil as shown in Figure 17.3. Magnetic field of both the groups is calculated on their axis first, integrating the eqn. (17.1) to eqn. (17.3) for half cycle only.

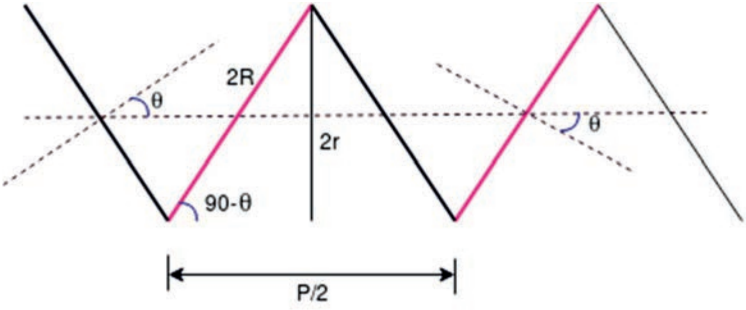

Figure 17.3. Simplification of helical coil

Axial and radial components of both the groups along their axis can be transferred to axial and radial components of full coil taking θ into account. P and r are the pitch and radius of the coil.

$$\tan\theta = \frac{P}{4r} \tag{17.4}$$

$$R = r/\cos\theta \tag{17.5}$$

Some of the coil developed and integrated with several pulse power systems are mentioned as follows;

17.1.1. Magnetic Coil for X-band BWO

Figure 17.4 shows the photo of the coil. Magnetic field profile at coil axis is shown in Figure 17.5. Specifications of the coil are given in Table 17.1.

Table 17.1. Specifications of coil for X-band BWO.

Coil configuration	Solenoid with extra turns at the ends
Average magnetic field at axis	0.9 T@1 kA
Rated current	5 kA pulse (<10ms)
Conductor material	copper
Cross section	6 mm x 3 mm
Insulation thickness on the coil	0.5 mm
Length of the coil without former	238 mm
Number of layers in solenoid	5
Number of turns in each layers	34
Extra layers at each ends	5
Number of turns in each extra layer	4
Coil inductance LC	1 mH
Coil Resistance RC at 1kHz	98 mΩ

Figure 17.4. Magnetic coil for X-band BWO.

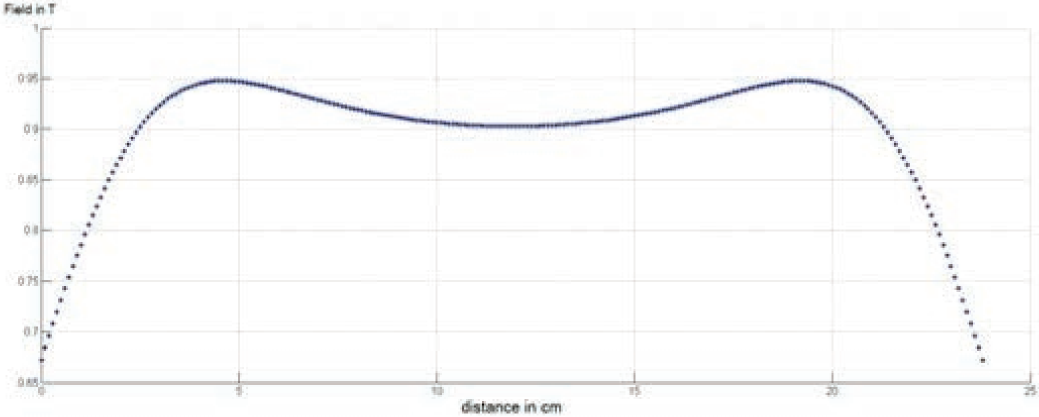

Figure 17.5. Magnetic field profile of X-band BWO coil for 1 kA current.

17.1.2. Magnetic Coil for S-band BWO

A magnetic coil for 1 Tesla pulsed field is designed and developed for S-band BWO. It consists of two coils, placed at 50 mm distance and connected in series. Figure 17.6 and Figure 17.7 show the schematic and photo of the coil. Magnetic field profile at axis of the coil is shown in Figure 17.8. Specifications of the coil are given in Table 17.2.

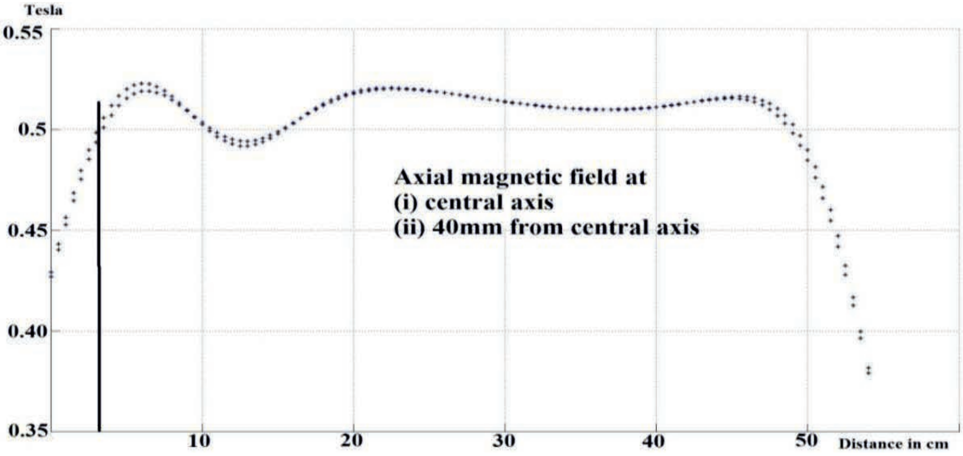

Figure 17.6. Magnetic field profile of S-band BWO for 1kA current.

Table 17.2. Specifications of coil for S-band BWO.

Coil configuration	Hybrid
Average magnetic field at axis	0.5 T/1 kA
Rated current	2 kA pulse (<10ms)
Conductor material	Copper
Wire gauge	SWG16
Insulation	0.1 mm enamel
Length of the coil 1 (bigger one)	410 mm
Inner diameter of coil 1	165 mm
Number of layers in coil 1	18
Turns in each layer in coil 1	32
Extra layers at end of coil 1	6
Turns in extra layers in coil 1	16
Length of the coil 2	80 mm
Inner diameter of coil 2	220 mm
Number of layers in coil 1	18
Turns in each layer in coil 1	32
Distance between both the coils	50 mm
Total inductance of the coil	138 mH
Total resistance of the coil at 1 kHz	2.9 mΩ
Weight of the coil	62 kg

17.1.3. Magnetic Coil for Relativistic Magnetron

Magnetic coil for relativistic magnetron was developed for DC magnetic field of 0.5 T. Two water cooled coils are placed at 50mm distance and connected in series. Figure 17.7 and Figure 17.8 show the image and drawing of side view. Specifications of the coil are given in Table 17.3.

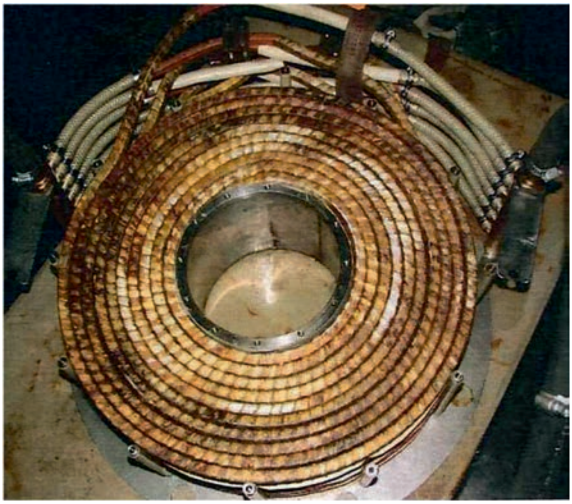

Figure 17.7. Coil for Relativistic magnetron.

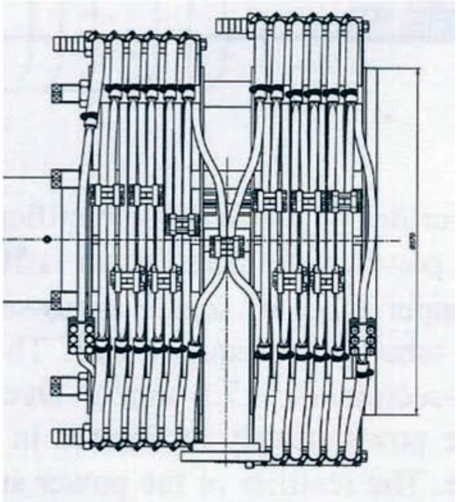

Figure 17.8. Drawing of magnetic coil's side view.

Table 17.3. Specifications of coil for Relativistic Magnetron

Conductor	copper
Cross section	12.5 mm x 12.5 mm with 8mm bore diameter
Magnetic field	0.5 T/kA over 60mm length
Rated current	1 kA
Number of layers in each coil	10
Number of turns in each layer	6
Distance between the coils	50 mm
Inner diameter of the coil	200 mm

17.2. Design of Capacitor Bank for Pulse Magnetic Field

For pulse magnetic field, magnetic coil is energized, discharging an energy storage capacitor. As energy storage capacitor is formed by number of capacitors, it is also called energy storage capacitor. Schematic of electrical diagram for the scheme is shown in Figure 17.9.

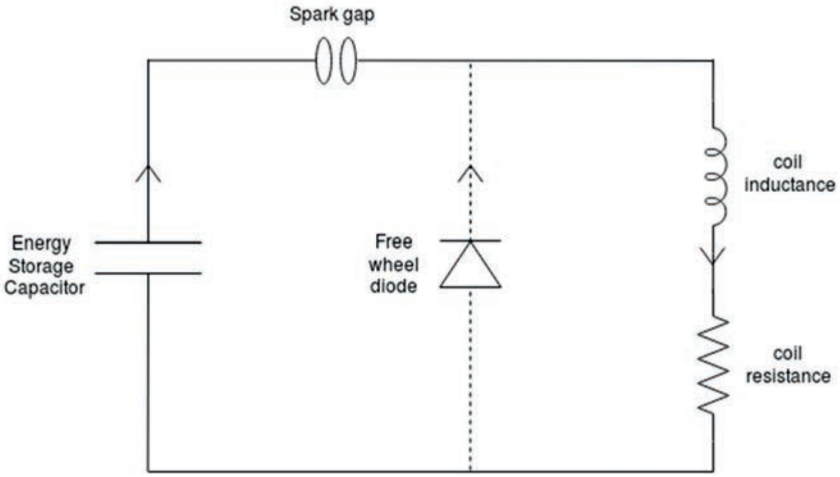

Figure 17.9. Electrical circuit diagram for pulse magnetic field.

Typical waveform of coil current and capacitor voltage during the discharge is shown in Figure 17.10.

Design of energy storage capacitor has following two objectives,

(i) To energize magnetic coil at required current.

(ii) Minimize size of the capacitor unit.

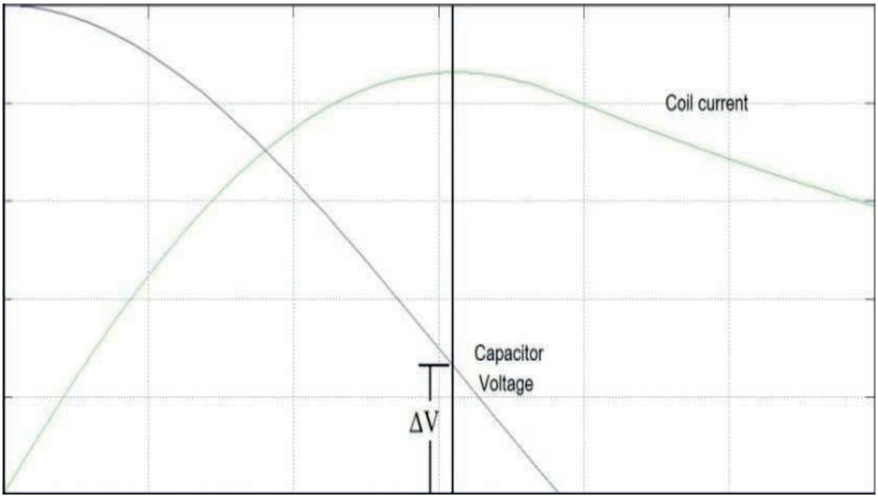

Figure 17.10. Typical waveform of coil current and capacitor voltage.

Energy stored in the magnetic coil having coil inductance L at required current I_R is given by,

$$E_R = \frac{1}{2} L I_R^{\,2} \tag{17.6}$$

Energy stored in the capacitor is maximum at initial voltage V_M and is given by,

$$E_M = \frac{1}{2} C V_M{}^2 \tag{17.7}$$

Maximum value of V_M is limited by breakdown strength E_b of the dielectric. If dielectric constant, area of dielectric and gap between dielectric film is d;

$$V_M = E_b d \tag{17.8}$$

$$C = \frac{\varepsilon A}{d} \tag{17.9}$$

From Eqs. (17.6), (17.7) and (17.8),

$$Ad = \frac{2E_M}{\varepsilon E^2} \tag{17.10}$$

It is clear from the Eq. (17.10) that for a given dielectric, required energy E_M should be minimum for minimum size of the capacitor. Relation between E_R and E_M are derived analysing the voltage and current waveform as follows. Coil current and capacitor voltage are given by Eqs. (17.11) and (17.12) respectively after the spark gap fired till the capacitor is discharged fully at time t_c.

$$I = \frac{V e^{-\alpha t} \sin \omega_d t}{\omega_d L} \tag{17.11}$$

$$V_C = \frac{\omega_n V_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t + \theta) \tag{17.12}$$

$$\text{Here } \alpha = \frac{R}{2L} \tag{17.13}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{17.14}$$

$$t_c = \frac{\pi - \theta}{\omega_d} \tag{17.15}$$

$$\sin\theta = \frac{\omega_d}{\omega_n} \tag{17.16}$$

$$\cos\theta = \frac{\alpha}{\omega_n} \tag{17.17}$$

Maximum coil current I_P and its instant t_p is given by Eqs. (17.16) and (17.17).

$$I_P = V_0 e^{-\alpha t_P} \sqrt{\frac{C}{L}} \tag{17.18}$$

$$t_P = \frac{\sin^{-1}\left(\frac{\omega_d}{\omega_n}\right)}{\omega_d} \tag{17.19}$$

Coil current starts to flow through free wheel diode as per Eq. (17.20). Current at this moments starts to decreases exponentially from I_C.

$$I = I_C e^{-\alpha(t-t_c)} \tag{17.20}$$

$$I_C = V_0 e^{-\alpha t_c} \sqrt{\frac{C}{L}} \tag{17.21}$$

Eqs. (17.21) and (17.22) give the relation between E_R and E_M for required coil current at the instants t_p and t_c respectively. EM is relatively lower for t_p then t_c but required current at t_c is advantageous as rate of change in the current is relatively lower after t_c.

$$\left(E_M\right)_{t_P} = E_R e^{2\alpha t_P} \tag{17.22}$$

$$\left(E_M\right)_{t_C} = E_R e^{2\alpha t_C} \tag{17.23}$$

Figure 17.11 shows two curves showing relation between capacitance with charging voltage and energy transfer ratio for peak coil current. Coil parameters and required current are of X-band BWO coil. Capacitor voltage is preferred to be low for safety and insulation point of view. It can be inferred from the curves that high capacitance results into low initial voltage. But energy transfer ratio decreases, which results into bigger size of the bank. Optimization of size and insulation are the basis of capacitor bank's parameter.

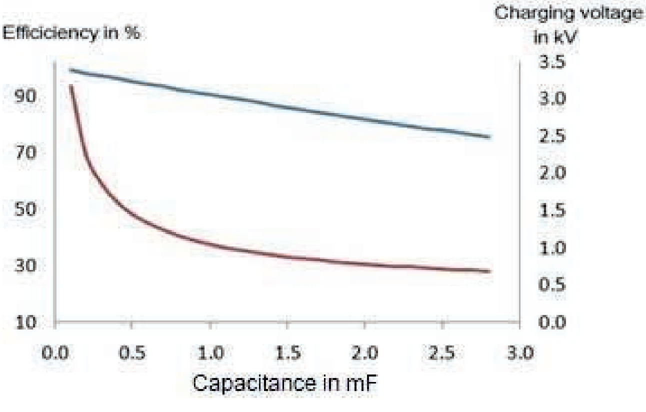

Figure 17.11. Energy transfer efficiency and charging voltage against capacitance.

Figure 17.12 shows the photo of capacitor bank designed and developed for the X-band BWO. It consists of 24 capacitors, which are connected in configuration of 6 series and 4 parallel. Each one is of 1.67 mF and 1.1 kV rated voltage. Net capacitance and rated voltage of the capacitor bank is 1.1 mF and 6.6 kV respectively.

Figure 17.12. 1.1 mF, 6.6 kV capacitor bank.

References

[1] Evandro Paese., Martin Geier., et al.“ Mathematical Modeling of an Electromagnetic Forming System with Flat Spiral Coils as Actuator” Journal of the Brazilian Society of Mechanical Sciences and Engineering 33(3), 2011.

[2] V K Sharma., A S Patel., et al. “Design and analysis of magnetic coil for relativistic magnetron”International Symposium on Discharges and Electrical Insulation in Vacuum 2014.